Mathematics — HSC Reference Sheet

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Measurement

Area of a sector

$$A = \tfrac{1}{2}r^2\theta$$

Arc length

$$l = r\theta$$

Surface area of a cone (lateral)

$$S = \pi r l$$

Surface area of a sphere

$$S = 4\pi r^2$$

Volume of a cone

$$V = \tfrac{1}{3}\pi r^2 h$$

Volume of a sphere

$$V = \tfrac{4}{3}\pi r^3$$

Volume of a pyramid

$$V = \tfrac{1}{3}Ah$$

Financial Mathematics

Future value of a compound interest investment

$$FV = PV(1+r)^n$$

Present value of a compound interest investment

$$PV = FV(1+r)^{-n}$$

Future value of an annuity (regular contributions)

$$FV = C\,\frac{(1+r)^n - 1}{r}$$

Present value of an annuity

$$PV = C\,\frac{1-(1+r)^{-n}}{r}$$

Sequences and Series

Arithmetic sequence — nth term

$$T_n = a + (n-1)d$$

Arithmetic series — sum of n terms

$$S_n = \frac{n}{2}(a+l) = \frac{n}{2}[2a + (n-1)d]$$

Geometric sequence — nth term

$$T_n = ar^{n-1}$$

Geometric series — sum of n terms

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$$

Infinite geometric series — limiting sum

$$S = \frac{a}{1-r}, \quad |r| < 1$$

Logarithmic, Exponential and Indices

Change of base formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Logarithm product law

$$\log_a(xy) = \log_a x + \log_a y$$

Logarithm quotient law

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Logarithm power law

$$\log_a(x^n) = n\log_a x$$

Logarithm definition / inverse relationship

$$\log_a a^x = x \quad \text{and} \quad a^{\log_a x} = x$$

Index law — product

$$a^m \cdot a^n = a^{m+n}$$

Index law — quotient

$$\frac{a^m}{a^n} = a^{m-n}$$

Index law — power of a power

$$(a^m)^n = a^{mn}$$

Derivative of a^x (general exponential)

$$\frac{d}{dx}(a^x) = a^x \ln a$$

Trigonometry

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$A = \tfrac{1}{2}ab\sin C$$

Exact trigonometric ratios — 30°, 45°, 60°

$$\sin 30^\circ = \tfrac{1}{2}, \quad \cos 30^\circ = \tfrac{\sqrt{3}}{2}, \quad \tan 30^\circ = \tfrac{1}{\sqrt{3}}$$

Pythagorean identity

$$\sin^2\theta + \cos^2\theta = 1$$

Compound angle — sine

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

Compound angle — cosine

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

Compound angle — tangent

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double angle — sine

$$\sin 2A = 2\sin A\cos A$$

Double angle — cosine (three forms)

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Double angle — tangent

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Products to sums (product formulae)

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

t-formulae (half-angle substitution $t = \tan(\theta/2)$)

$$\sin\theta = \frac{2t}{1+t^2}, \quad \cos\theta = \frac{1-t^2}{1+t^2}, \quad \tan\theta = \frac{2t}{1-t^2}$$

Calculus — Differentiation

Power rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Product rule

$$\frac{d}{dx}(uv) = u'v + uv'$$

Quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

Chain rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

Derivative of $\sin x$

$$\frac{d}{dx}(\sin x) = \cos x$$

Derivative of $\cos x$

$$\frac{d}{dx}(\cos x) = -\sin x$$

Derivative of $\tan x$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

Derivative of e^x

$$\frac{d}{dx}(e^x) = e^x$$

Derivative of $\ln x$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$$

Calculus — Standard Integrals

Power rule for integration

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Integral of $1/x$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

Integral of e^{ax}

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$$

Integral of $\cos ax$

$$\int \cos(ax) \, dx = \frac{1}{a} \sin(ax) + C$$

Integral of $\sin ax$

$$\int \sin(ax) \, dx = -\frac{1}{a} \cos(ax) + C$$

Integral of $\sec^2 ax$

$$\int \sec^2(ax) \, dx = \frac{1}{a} \tan(ax) + C$$

Integral giving arcsin (inverse sine)

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \quad |x| < a$$

Integral giving arctan (inverse tangent)

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

Integral of a^x (general exponential)

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

Log-integral (reverse chain rule for $\ln$)

$$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$$

Probability and Statistics

Complement rule

$$P(\bar{A}) = 1 - P(A)$$

Addition rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Mean (expected value) of a discrete random variable

$$\mu = E(X) = \sum x_i P(X = x_i)$$

Variance and standard deviation of a discrete random variable

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$$

Binomial distribution

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

Normal distribution and z-score

$$z = \frac{x - \mu}{\sigma}$$

Combinatorics (Ext 1)

Permutations (ordered selections)

$${}^nP_r = \frac{n!}{(n-r)!}$$

Combinations (unordered selections)

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

Ext 1 — Further Calculus and Functions (Ext 1)

Derivative of $\sin^{-1} x$

$$\frac{d}{dx}\left(\sin^{-1}\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$$

Derivative of $\cos^{-1} x$

$$\frac{d}{dx}\left(\cos^{-1}\frac{x}{a}\right) = -\frac{1}{\sqrt{a^2 - x^2}}$$

Derivative of $\tan^{-1} x$

$$\frac{d}{dx}\left(\tan^{-1}\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$$

Integration by substitution

$$\int f(g(x))\, g'(x) \, dx = F(g(x)) + C$$

Ext 1 — Vectors (2D) (Ext 1)

Magnitude of a vector

$$|\mathbf{u}| = \sqrt{x^2 + y^2}$$

Dot product (scalar product)

$$\mathbf{u} \cdot \mathbf{v} = x_1x_2 + y_1y_2 = |\mathbf{u}||\mathbf{v}| \cos\theta$$

Scalar projection of $\mathbf{u}$ onto $\mathbf{v}$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|}$$

Ext 1 — Projectile Motion (Ext 1)

Horizontal displacement

$$x = V\cos\alpha \cdot t$$

Vertical displacement

$$y = V\sin\alpha \cdot t - \tfrac{1}{2}gt^2$$

Range on a horizontal plane

$$R = \frac{V^2 \sin 2\alpha}{g}$$

Ext 2 — Complex Numbers (Ext 2)

Modulus-argument (polar) form

$$z = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$$

Multiplication of complex numbers in polar form

$$r_1\,\text{cis}\,\theta_1 \times r_2\,\text{cis}\,\theta_2 = r_1r_2\,\text{cis}(\theta_1 + \theta_2)$$

De Moivre's theorem

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Ext 2 — Further Integration (Ext 2)

Integration by parts

$$\int u\,dv = uv - \int v\,du$$

Partial fractions

$$\frac{P(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

t-substitution for integrals ($t = \tan(x/2)$)

$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2}{1+t^2}\,dt$$

Ext 2 — Vectors in 3D, Mechanics and SHM (Ext 2)

3D vector magnitude

$$|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$$

3D dot product and angle

$$\mathbf{u} \cdot \mathbf{v} = x_1x_2 + y_1y_2 + z_1z_2 = |\mathbf{u}||\mathbf{v}| \cos\theta$$

Simple harmonic motion — equation

$$\ddot{x} = -n^2x$$

SHM — displacement, velocity, acceleration solutions

$$x = a\cos(nt + \alpha) \quad \text{or} \quad x = a\sin(nt + \alpha)$$

SHM — velocity in terms of displacement

$$v^2 = n^2(a^2 - x^2)$$

Resisted motion — Newton's second law form

$$m\ddot{x} = F_{\text{net}}$$