

Constants & Data

Avogadro's constant

$$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

Universal gas constant

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

Molar volume of an ideal gas at STP (0 °C, 100 kPa)

$$V_m = 22.71 \text{ L mol}^{-1}$$

Molar volume of an ideal gas at 25 °C, 100 kPa

$$V_m = 24.79 \text{ L mol}^{-1}$$

Ionic product of water

$$K_w = 1.0 \times 10^{-14} \text{ mol}^2 \text{ L}^{-2}$$

Specific heat capacity of water

$$c = 4.18 \text{ J g}^{-1} \text{ K}^{-1}$$

Faraday constant

$$F = 96\,485 \text{ C mol}^{-1}$$

Mole & Stoichiometry

Moles from mass

$$n = \frac{m}{M}$$

Moles from number of particles

$$n = \frac{N}{N_A}$$

Moles from volume (solutions)

$$n = cV$$

Gases

Ideal gas law

$$PV = nRT$$

Molar volume (gas volume)

$$V = nV_m$$

Solutions & Concentration

Molar concentration

$$c = \frac{n}{V}$$

Dilution formula

$$c_1V_1 = c_2V_2$$

Parts per million (mass/mass)

$$\text{ppm} = \frac{m_{\text{solute}}}{m_{\text{solution}}} \times 10^6$$

Energetics & Calorimetry

Heat transferred (calorimetry)

$$q = mc\Delta T$$

Molar enthalpy of reaction

$$\Delta H = -\frac{q}{n}$$

Hess's Law

$$\Delta H_{\text{rxn}} = \sum \Delta H_{\text{products}} - \sum \Delta H_{\text{reactants}}$$

Equilibrium & Acids/Bases

Equilibrium constant expression

$$K_{eq} = \frac{[C]^c[D]^d}{[A]^a[B]^b}$$

Reaction quotient

$$Q = \frac{[C]^c[D]^d}{[A]^a[B]^b}$$

pH definition

$$\text{pH} = -\log_{10}[\text{H}^+]$$

pOH and the water relationship

$$\text{pH} + \text{pOH} = 14$$

Ionic product of water

$$K_w = [\text{H}^+][\text{OH}^-] = 1.0 \times 10^{-14}$$

Acid dissociation constant

$$K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]}$$

pKa

$$\text{p}K_a = -\log_{10} K_a$$

Electrochemistry

Charge transferred in electrolysis

$$Q = It$$

Moles of electrons (Faraday's law)

$$n_e = \frac{Q}{F}$$

Mass deposited in electrolysis

$$m = nM$$

Measurement

Area of a sector

$$A = \frac{1}{2}r^2\theta$$

Arc length

$$l = r\theta$$

Surface area of a cone (lateral)

$$S = \pi rl$$

Surface area of a sphere

$$S = 4\pi r^2$$

Volume of a cone

$$V = \frac{1}{3}\pi r^2 h$$

Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

Volume of a pyramid

$$V = \frac{1}{3}Ah$$

Financial Mathematics

Future value of a compound interest investment

$$FV = PV(1 + r)^n$$

Present value of a compound interest investment

$$PV = FV(1 + r)^{-n}$$

Future value of an annuity (regular contributions)

$$FV = C \frac{(1 + r)^n - 1}{r}$$

Present value of an annuity

$$PV = C \frac{1 - (1 + r)^{-n}}{r}$$

Sequences and Series

Arithmetic sequence — nth term

$$T_n = a + (n - 1)d$$

Arithmetic series — sum of n terms

$$S_n = \frac{n}{2}(a + l) = \frac{n}{2}[2a + (n - 1)d]$$

Geometric sequence — nth term

$$T_n = ar^{n-1}$$

Geometric series — sum of n terms

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}$$

Infinite geometric series — limiting sum

$$S = \frac{a}{1 - r}, \quad |r| < 1$$

Logarithmic, Exponential and Indices

Change of base formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Logarithm product law

$$\log_a(xy) = \log_a x + \log_a y$$

Logarithm quotient law

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Logarithm power law

$$\log_a(x^n) = n \log_a x$$

Logarithm definition / inverse relationship

$$\log_a a^x = x \quad \text{and} \quad a^{\log_a x} = x$$

Index law — product

$$a^m \cdot a^n = a^{m+n}$$

Index law — quotient

$$\frac{a^m}{a^n} = a^{m-n}$$

Index law — power of a power

$$(a^m)^n = a^{mn}$$

Derivative of a^x (general exponential)

$$\frac{d}{dx}(a^x) = a^x \ln a$$

Trigonometry

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Area of a triangle

$$A = \frac{1}{2}ab \sin C$$

Exact trigonometric ratios — 30°, 45°, 60°

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Pythagorean identity

$$\sin^2 \theta + \cos^2 \theta = 1$$

Compound angle — sine

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

Compound angle — cosine

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Compound angle — tangent

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double angle — sine

$$\sin 2A = 2 \sin A \cos A$$

Double angle — cosine (three forms)

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

Double angle — tangent

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Products to sums (product formulae)

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

t-formulae (half-angle substitution $t = \tan(\theta/2)$)

$$\sin \theta = \frac{2t}{1+t^2}, \quad \cos \theta = \frac{1-t^2}{1+t^2}, \quad \tan \theta = \frac{2t}{1-t^2}$$

Calculus — Differentiation

Power rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Product rule

$$\frac{d}{dx}(uv) = u'v + uv'$$

Quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Derivative of $\sin x$

$$\frac{d}{dx}(\sin x) = \cos x$$

Derivative of $\cos x$

$$\frac{d}{dx}(\cos x) = -\sin x$$

Derivative of $\tan x$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

Derivative of e^x

$$\frac{d}{dx}(e^x) = e^x$$

Derivative of $\ln x$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$$

Calculus — Standard Integrals

Power rule for integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Integral of $1/x$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Integral of e^{ax}

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

Integral of $\cos ax$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + C$$

Integral of $\sin ax$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + C$$

Integral of $\sec^2 ax$

$$\int \sec^2(ax) dx = \frac{1}{a}\tan(ax) + C$$

Integral giving arcsin (inverse sine)

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \quad |x| < a$$

Integral giving arctan (inverse tangent)

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Integral of a^x (general exponential)

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

Log-integral (reverse chain rule for ln)

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

Probability and Statistics

Complement rule

$$P(\bar{A}) = 1 - P(A)$$

Addition rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Mean (expected value) of a discrete random variable

$$\mu = E(X) = \sum x_i P(X = x_i)$$

Variance and standard deviation of a discrete random variable

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

Binomial distribution

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

Normal distribution and z-score

$$z = \frac{x - \mu}{\sigma}$$

Combinatorics (Ext 1)

Permutations (ordered selections)

$${}^n P_r = \frac{n!}{(n-r)!}$$

Combinations (unordered selections)

$$\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

Ext 1 — Further Calculus and Functions (Ext 1)

Derivative of $\sin^{-1} x$

$$\frac{d}{dx} \left(\sin^{-1} \frac{x}{a} \right) = \frac{1}{\sqrt{a^2 - x^2}}$$

Derivative of $\cos^{-1} x$

$$\frac{d}{dx} \left(\cos^{-1} \frac{x}{a} \right) = -\frac{1}{\sqrt{a^2 - x^2}}$$

Derivative of $\tan^{-1} x$

$$\frac{d}{dx} \left(\tan^{-1} \frac{x}{a} \right) = \frac{a}{a^2 + x^2}$$

Integration by substitution

$$\int f(g(x)) g'(x) dx = F(g(x)) + C$$

Ext 1 — Vectors (2D) (Ext 1)

Magnitude of a vector

$$|\mathbf{u}| = \sqrt{x^2 + y^2}$$

Dot product (scalar product)

$$\mathbf{u} \cdot \mathbf{v} = x_1x_2 + y_1y_2 = |\mathbf{u}||\mathbf{v}| \cos \theta$$

Scalar projection of \mathbf{u} onto \mathbf{v}

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|}$$

Ext 1 — Projectile Motion (Ext 1)

Horizontal displacement

$$x = V \cos \alpha \cdot t$$

Vertical displacement

$$y = V \sin \alpha \cdot t - \frac{1}{2}gt^2$$

Range on a horizontal plane

$$R = \frac{V^2 \sin 2\alpha}{g}$$

Ext 2 — Complex Numbers (Ext 2)

Modulus-argument (polar) form

$$z = r(\cos \theta + i \sin \theta) = r \text{cis } \theta$$

Multiplication of complex numbers in polar form

$$r_1 \text{cis } \theta_1 \times r_2 \text{cis } \theta_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$$

De Moivre's theorem

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

Euler's formula

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Ext 2 — Further Integration (Ext 2)

Integration by parts

$$\int u dv = uv - \int v du$$

Partial fractions

$$\frac{P(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

t-substitution for integrals ($t = \tan(x/2)$)

$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2}{1+t^2} dt$$

Ext 2 — Vectors in 3D, Mechanics and SHM (Ext 2)

3D vector magnitude

$$|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$$

3D dot product and angle

$$\mathbf{u} \cdot \mathbf{v} = x_1x_2 + y_1y_2 + z_1z_2 = |\mathbf{u}||\mathbf{v}| \cos \theta$$

Simple harmonic motion — equation

$$\ddot{x} = -n^2x$$

SHM — displacement, velocity, acceleration solutions

$$x = a \cos(nt + \alpha) \quad \text{or} \quad x = a \sin(nt + \alpha)$$

SHM — velocity in terms of displacement

$$v^2 = n^2(a^2 - x^2)$$

Resisted motion — Newton's second law form

$$m\ddot{x} = F_{\text{net}}$$

Measurement

Area of a sector

$$A = \frac{\theta}{360} \pi r^2$$

Arc length

$$l = \frac{\theta}{360} \times 2\pi r$$

Pythagoras' theorem

$$c^2 = a^2 + b^2$$

Right-triangle trigonometry (SOH CAH TOA)

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos \theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Area of a triangle (two sides, included angle)

$$A = \frac{1}{2} ab \sin C$$

Surface area of a sphere

$$SA = 4\pi r^2$$

Volume of a sphere

$$V = \frac{4}{3} \pi r^3$$

Volume of a cone

$$V = \frac{1}{3} \pi r^2 h$$

Volume of a prism / cylinder

$$V = Ah$$

Simpson's rule

$$A \approx \frac{h}{2} (d_f + d_i)$$

Limits of accuracy

$$\text{Absolute error} = \frac{1}{2} \times \text{precision}$$

Area of a trapezium

$$A = \frac{h}{2} (a + b)$$

Financial Mathematics

Simple interest

$$I = Prn$$

Compound interest — future value

$$FV = PV(1 + r)^n$$

Compound interest — present value

$$PV = FV(1 + r)^{-n}$$

Straight-line (flat-rate) depreciation

$$S = V_0 - Dn$$

Declining-balance (reducing-value) depreciation

$$S = V_0(1 - r)^n$$

Future value of an annuity

$$FV = M \frac{(1 + r)^n - 1}{r}$$

Present value of an annuity

$$PV = M \frac{1 - (1 + r)^{-n}}{r}$$

Statistical Analysis

Mean (average)

$$\bar{x} = \frac{\sum x}{n}$$

Standard deviation (population, σ)

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

z-score (standardised score)

$$z = \frac{x - \mu}{\sigma}$$

Empirical rule (68-95-99.7 rule)

$$68\%, 95\%, 99.7\%$$

Least-squares regression line

$$\hat{y} = a + bx$$

Pearson's correlation coefficient

$$-1 \leq r \leq 1$$

Outlier (IQR rule)

$$x < Q_1 - 1.5 \text{ IQR} \text{ or } x > Q_3 + 1.5 \text{ IQR}$$

Algebra & Linear/Non-linear Relationships

Gradient of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Gradient-intercept form of a line

$$y = mx + b$$

Point-gradient form / general linear equation

$$y - y_1 = m(x - x_1)$$

Exponential model

$$y = a \cdot b^x$$

Networks

Euler's formula for connected planar graphs

$$V - E + F = 2$$

Minimum spanning tree (Prim's / Kruskal's)

$$\text{Total weight} = \sum (\text{weights of selected edges})$$

Shortest path (Dijkstra's algorithm concept)

$$d(v) = \min(d(u) + w(u, v))$$

Critical path analysis — float time

$$\text{Float} = \text{LST} - \text{EST}$$

Constants & Data

Speed of light in vacuum

$$c = 3.00 \times 10^8 \text{ m s}^{-1}$$

Charge on an electron

$$e = 1.602 \times 10^{-19} \text{ C}$$

Mass of an electron

$$m_e = 9.109 \times 10^{-31} \text{ kg}$$

Mass of a proton

$$m_p = 1.673 \times 10^{-27} \text{ kg}$$

Mass of a neutron

$$m_n = 1.675 \times 10^{-27} \text{ kg}$$

Planck's constant

$$h = 6.626 \times 10^{-34} \text{ J s}$$

Universal gravitational constant

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

Acceleration due to gravity at Earth's surface

$$g = 9.8 \text{ m s}^{-2}$$

Permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1} \text{ (N A}^{-2}\text{)}$$

Permittivity of free space

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F m}^{-1} \text{ (C}^2 \text{ N}^{-1} \text{ m}^{-2}\text{)}$$

Mass of Earth

$$M_E = 6.0 \times 10^{24} \text{ kg}$$

Radius of Earth

$$R_E = 6.371 \times 10^6 \text{ m}$$

Speed of sound in air

$$v_{\text{sound}} = 340 \text{ m s}^{-1}$$

Wien's displacement constant

$$b = 2.898 \times 10^{-3} \text{ m K}$$

Kinematics

First kinematic equation (velocity–time)

$$v = u + at$$

Second kinematic equation (displacement–time)

$$s = ut + \frac{1}{2}at^2$$

Third kinematic equation (velocity–displacement)

$$v^2 = u^2 + 2as$$

Displacement (average velocity form)

$$s = \frac{(u + v)}{2} t$$

Dynamics & Forces

Newton's second law

$$F_{\text{net}} = ma$$

Linear momentum

$$p = mv$$

Impulse–momentum theorem

$$J = F\Delta t = \Delta p$$

Work done by a constant force

$$W = Fs \cos \theta$$

Power

$$P = \frac{W}{t} = Fv$$

Kinetic energy

$$E_k = \frac{1}{2}mv^2$$

Gravitational potential energy

$$E_p = mgh$$

Circular & Projectile Motion

Centripetal acceleration

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

Centripetal force

$$F_c = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2}$$

Projectile horizontal component

$$x = v_x t = v_0 \cos \theta \cdot t$$

Projectile vertical component

$$y = v_0 \sin \theta \cdot t - \frac{1}{2}gt^2$$

Gravitation & Orbits

Newton's law of universal gravitation

$$F_g = \frac{Gm_1m_2}{r^2}$$

Gravitational potential energy (general)

$$U = -\frac{Gm_1m_2}{r}$$

Orbital velocity

$$v = \sqrt{\frac{GM}{r}}$$

Kepler's third law

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Waves & Sound

Wave speed

$$v = f\lambda$$

Doppler effect

$$f' = f \frac{v_w + v_o}{v_w - v_s}$$

Period–frequency relationship

$$T = \frac{1}{f}$$

Electricity & Magnetism

Charge and current

$$q = It$$

Work on a charge

$$W = qV$$

Coulomb's law

$$F_e = \frac{kq_1q_2}{r^2} = \frac{q_1q_2}{4\pi\epsilon_0r^2}$$

Electric field strength

$$E = \frac{F}{q} = \frac{kQ}{r^2}$$

Voltage and resistance (Ohm's law)

$$V = IR$$

Electrical power

$$P = VI = I^2R = \frac{V^2}{R}$$

Force on a current-carrying conductor

$$F = BIl \sin \theta$$

Force on a moving charge in a magnetic field

$$F = qvB \sin \theta$$

Electromagnetism & Induction

Magnetic flux

$$\Phi = BA \cos \theta$$

Faraday's law (induced EMF)

$$\mathcal{E} = -N \frac{\Delta\Phi}{\Delta t}$$

Transformer equation

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

Light & Quantum Physics

Photon energy

$$E = hf = \frac{hc}{\lambda}$$

Photoelectric effect (Einstein's equation)

$$E_k^{\max} = hf - W = hf - hf_0$$

de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Wien's displacement law

$$\lambda_{\max} = \frac{b}{T}$$

Special relativity — time dilation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma t_0$$

Special relativity — length contraction

$$L = \frac{L_0}{\gamma} = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Nuclear Physics & the Standard Model

Mass–energy equivalence

$$E = mc^2$$

Radioactive decay / half-life

$$N(t) = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

Photon momentum

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

Relativistic energy–momentum relation

$$E^2 = (pc)^2 + (m_0c^2)^2$$

