

INTUITION EDUCATION

HSC 2026 · PREDICTED PRACTICE PAPER

2026 HSC Mathematics Extension 2

*Answers & marking notes***Paper total: 100 marks**

Sit the paper first. These are worked answers with the marking notes behind them — what earns the mark, and the traps the marking centre has flagged in past years. Where a note names a band, it describes what a top-band response does, not a guaranteed mark.

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Answers & marking notes

Section I — correct responses

Q	1	2	3	4	5	6	7	8	9	10
Answer	B	C	D	C	B	A	A	D	B	C

Brief reasons:

- Contrapositive negates **and swaps**: "not multiple of 3 $\Rightarrow$ not multiple of 6". A is the converse, C the inverse, D a (false) conjunction. **B**
- Negating $\forall \exists$ gives $\exists \forall$: there is some x that is no cube. A negates only the inner clause; D over-negates. **C**
- $-1 - \sqrt{3}i$ lies in the third quadrant with reference angle $\frac{\pi}{3}$; the principal argument in $(-\pi, \pi]$ is $-\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$ (B is the non-principal $\frac{4\pi}{3}$). **D**
- Direction $\overrightarrow{AB} = (2, 1, -2)$ through point A . A and B use a position vector as the direction; D uses $B - A$ as the *point*. **C**
- $a = v \frac{dv}{dx} = (x^2 + 1)(2x)$; at $x = 1$: $2 \times 2 = 4$. (A is the omitted- v error $\frac{dv}{dx} = 2$.) **B**
- $v^2 = 16 - (x - 2)^2$: $n = 1$, centre 2, amplitude 4. (B is the read-the-constant-term error $A^2 = 12$; D is the centre.) **A**
- Conjugate $2 - i$ is also a zero; sum of zeros = 7, so the third is $7 - 4 = 3$ (or $15 \div |2 + i|^2 = 3$ from the product). **A**
- Irreducible quadratic $x^2 + 4$ needs a linear numerator $Bx + C$; it does not factor over the reals (B), and $\frac{A}{x-1}$ needs only a constant (C). **D**
- $1 + i = \sqrt{2} \operatorname{cis} \frac{\pi}{4}$, so $(1 + i)^{10} = 2^5 \operatorname{cis} \frac{5\pi}{2} = 32 \operatorname{cis} \frac{\pi}{2} = 32i$. **B**
- Equidistant from 1 and $-i$: the perpendicular bisector of the segment joining $(1, 0)$ and $(0, -1)$, which passes through the origin with gradient -1 : the line $y = -x$. **C**

Question 11

(a)(i) $\bar{z}w = (3 + i)(1 + 2i) = 3 + 6i + i - 2 = 1 + 7i$.

(a)(ii) $\frac{3 - i}{1 + 2i} \cdot \frac{1 - 2i}{1 - 2i} = \frac{3 - 6i - i - 2}{5} = \frac{1 - 7i}{5} = \frac{1}{5} - \frac{7}{5}i$. (1 mark multiplying by the conjugate, 1 mark answer.)

(b) $|\sqrt{3} - i| = 2$, $\arg = -\frac{\pi}{6}$, so $\sqrt{3} - i = 2e^{-i\pi/6}$. Then

$(\sqrt{3} - i)^7 = 2^7 e^{-7i\pi/6} = 128 \operatorname{cis} \frac{5\pi}{6} = 128 \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -64\sqrt{3} + 64i$. (1 mark exponential form with principal argument, 1 mark De Moivre, 1 mark exact Cartesian.)

(c) $a \cdot b = 2 - 3 - 4 = -5$; $|a| = 3$, $|b| = \sqrt{14}$. $\cos \theta = \frac{-5}{3\sqrt{14}}$, so $\theta \approx 116^\circ$. (Keep the negative cosine — the angle is obtuse.)

(d)(i) $\vec{AB} = (2, -1, 2): \vec{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}.$

(d)(ii) Require $(7, -1, 5) = (1 + 2\lambda, 2 - \lambda, -1 + 2\lambda)$: the first gives $\lambda = 3$, the second $\lambda = 3$, the third $\lambda = 3$ — one **consistent** parameter, so C lies on ℓ . (The mark is for checking all three components against a single λ .)

(e) Suppose $\log_2 3 = \frac{p}{q}$ with p, q positive integers (positive since $\log_2 3 > 0$). Then $2^{p/q} = 3$, so $2^p = 3^q$. But 2^p is even and 3^q is odd — a contradiction. Hence $\log_2 3$ is irrational. (1 mark stating the assumption in lowest form, 1 mark reaching $2^p = 3^q$, 1 mark exhibiting the parity contradiction **and writing the conclusion**.)

(f) $z = \pm 3i$.

Question 12

(a)(i) $2x^2 + x + 2 = A(x^2 + 4) + (Bx + C)(x - 1)$. At $x = 1$: $5 = 5A$, so $A = 1$. Comparing x^2 : $B = 1$; comparing constants: $4A - C = 2$, so $C = 2$. Thus $A = 1, B = 1, C = 2$.

(a)(ii) $\int \left(\frac{1}{x-1} + \frac{x+2}{x^2+4} \right) dx = \ln|x-1| + \frac{1}{2} \ln(x^2+4) + \tan^{-1} \frac{x}{2} + C$. (Absolute value on the first logarithm required.)

(b) $dx = \frac{2 dt}{1+t^2}$, $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$; limits $x = 0 \rightarrow t = 0$, $x = \frac{\pi}{2} \rightarrow t = 1$. The denominator becomes $\frac{2+2t}{1+t^2}$, so the integral is $\int_0^1 \frac{dt}{1+t} = \ln 2$. (1 mark substitution including **both limits**, 1 mark simplification, 1 mark $\ln 2$.)

(c)(i) $\ddot{x} = \frac{1}{2} \frac{d(v^2)}{dx} = \frac{1}{2} (18 - 18x) = -9(x-1)$, which is of the form $\ddot{x} = -n^2(x-c)$: SHM with centre $x = 1$ and $n = 3$, period $\frac{2\pi}{3}$.

(c)(ii) $v^2 = 36 - 9(x-1)^2$: amplitude $A = 2$; maximum speed $nA = 6 \text{ m s}^{-1}$ (at the centre).

(c)(iii) Maximum $|\ddot{x}| = n^2 A = 9 \times 2 = 18 \text{ m s}^{-2}$ (at the endpoints $x = -1, 3$).

(c)(iv) $4A = 8 \text{ m}$.

(d) Base case $n = 5$: $2^5 = 32 > 25 = 5^2$. Assume $2^k > k^2$ for some integer $k \geq 5$. Then $2^{k+1} = 2 \cdot 2^k > 2k^2$. Now for $k \geq 5$, $k^2 - 2k - 1 = (k-1)^2 - 2 > 0$, so $2k^2 > k^2 + 2k + 1 = (k+1)^2$. Hence $2^{k+1} > (k+1)^2$, and the result follows by induction. (The middle mark is for the auxiliary inequality $2k^2 > (k+1)^2$ with justification valid for $k \geq 5$ — the step where the assumption is *used*, not restated.)

Question 13

(a)(i) With $u = x^n$, $dv = e^x dx$, so $du = nx^{n-1} dx$, $v = e^x$:

$$I_n = [x^n e^x]_0^1 - n \int_0^1 x^{n-1} e^x dx = e - nI_{n-1} \quad (n \geq 1).$$

(1 mark naming u , dv , v , du ; 1 mark the parts step; 1 mark substituting the limits.)

(a)(ii) $I_0 = e - 1$; $I_1 = e - I_0 = 1$; $I_2 = e - 2I_1 = e - 2$; $I_3 = e - 3I_2 = 6 - 2e$.

(b) $|z - 2| \leq 2$ is the closed disc centre $(2, 0)$ radius 2 (solid boundary).

$|z| > |z - 2i| \iff x^2 + y^2 > x^2 + (y - 2)^2 \iff y > 1$: the open half-plane above the **dashed** line $y = 1$. The region is the part of the disc strictly above $y = 1$; the circle meets $y = 1$ at $(2 \pm \sqrt{3}, 1)$ (excluded endpoints, open dots). (1 mark disc with solid boundary, 1 mark dashed bisector $y = 1$ with correct side, 1 mark intersection points and correct shading.)

(c)(i) $(\sqrt{a} - \sqrt{b})^2 \geq 0 \Rightarrow a + b \geq 2\sqrt{ab} \Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$.

(c)(ii) By (i), $a + b \geq 2\sqrt{ab}$, $b + c \geq 2\sqrt{bc}$, $c + a \geq 2\sqrt{ca}$. All six quantities are positive, so the inequalities may be multiplied:

$$(a + b)(b + c)(c + a) \geq 8\sqrt{a^2b^2c^2} = 8abc.$$

(1 mark applying (i) three times, 1 mark justifying the multiplication by positivity, 1 mark completing; equality iff $a = b = c$. Start from the known truth — do not work backwards from the target.)

(d) General point $Q(1 + 2\lambda, \lambda, -1 + 2\lambda)$; $\overrightarrow{PQ} = (2\lambda - 2, \lambda + 1, 2\lambda - 3)$. Perpendicularity:

$\overrightarrow{PQ} \cdot (2, 1, 2) = 9\lambda - 9 = 0$, so $\lambda = 1$ and the closest point is $(3, 1, 1)$. Then $\overrightarrow{PQ} = (0, 2, -1)$ and the shortest distance is $\sqrt{5}$. (1 mark general point, 1 mark perpendicularity condition, 1 mark foot and distance.)

Question 14

(a)(i) $\overrightarrow{OD} = \frac{1}{3}\vec{a} + \frac{2}{3}\vec{b}$.

(a)(ii) X on OD : $\overrightarrow{OX} = t\left(\frac{1}{3}\vec{a} + \frac{2}{3}\vec{b}\right)$. X on BC : $\overrightarrow{OX} = \vec{b} + s\left(\frac{1}{2}\vec{a} - \vec{b}\right) = \frac{s}{2}\vec{a} + (1-s)\vec{b}$. Since $\vec{a}$ and $\vec{b}$ are non-zero and **non-parallel**, coefficients may be equated: $\frac{t}{3} = \frac{s}{2}$ and $\frac{2t}{3} = 1 - s$. Solving:

$t = \frac{3}{4}$, $s = \frac{1}{2}$. So $X = B + \frac{1}{2}(C - B)$ is the midpoint of BC , and $\overrightarrow{OX} = \frac{3}{4}\overrightarrow{OD}$ gives

$OX : XD = 3 : 1$. (1 mark the two expressions, 1 mark equating coefficients **with the non-parallel justification**, 1 mark both conclusions.)

(b)(i) $\cos 5\theta + i \sin 5\theta = (\cos \theta + i \sin \theta)^5$. The imaginary part of the binomial expansion (with $c = \cos \theta$, $s = \sin \theta$): $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$. Substituting $c^2 = 1 - s^2$:

$$\sin 5\theta = 5s(1 - s^2)^2 - 10s^3(1 - s^2) + s^5 = 16s^5 - 20s^3 + 5s.$$

(b)(ii) For $\theta = \frac{\pi}{5}$ and $\theta = \frac{2\pi}{5}$, $\sin 5\theta = \sin \pi = \sin 2\pi = 0$, so $s(16s^4 - 20s^2 + 5) = 0$ with $s = \sin \theta \neq 0$; hence both values satisfy $16x^4 - 20x^2 + 5 = 0$.

(b)(iii) Putting $y = x^2$: $16y^2 - 20y + 5 = 0$ has roots $y_1 = \sin^2 \frac{\pi}{5}$ and $y_2 = \sin^2 \frac{2\pi}{5}$ (distinct, as $0 < \sin \frac{\pi}{5} < \sin \frac{2\pi}{5}$). By Vieta, $y_1y_2 = \frac{5}{16}$, so $(\sin \frac{\pi}{5} \sin \frac{2\pi}{5})^2 = \frac{5}{16}$; both sines are positive, hence the product is $\frac{\sqrt{5}}{4}$. (The justification that the two roots of the quadratic are exactly these two values — and the positive-root choice — carries a mark.)

(c) Real coefficients $\Rightarrow 1 - 2i$ is also a zero. Sum of zeros = 5: third zero = $5 - (1 + 2i) - (1 - 2i) = 3$. (Check: $(1 + 4) \times 3 = 15$.) Other zeros: $1 - 2i$ and 3.

(d) Discriminant: $(3 + i)^2 - 4(2 + 2i) = 8 + 6i - 8 - 8i = -2i$. Seek $(x + iy)^2 = -2i$: $x^2 - y^2 = 0$, $2xy = -2$, giving $x = 1, y = -1$ (or $x = -1, y = 1$), so $\sqrt{-2i} = \pm(1 - i)$. Then

$$z = \frac{(3 + i) \pm (1 - i)}{2} = 2 \quad \text{or} \quad 1 + i.$$

(1 mark discriminant, 1 mark Cartesian square root, 1 mark both roots. Check: sum = $3 + i$, product = $2 + 2i$.)

Question 15

(a)(i) $v \frac{dv}{dx} = -(g + kv^2)$, so $x = - \int \frac{v dv}{g + kv^2} = -\frac{1}{2k} \ln(g + kv^2) + C$. At $x = 0, v = u$; at the top $v = 0$:

$$H = \frac{1}{2k} \ln \frac{g + ku^2}{g} = \frac{1}{2k} \ln \left(1 + \frac{ku^2}{g} \right).$$

(1 mark separating with the correct acceleration form, 1 mark the log integral with its constant, 1 mark applying both conditions to reach the given form — show every step in a "show that".)

(a)(ii) $\frac{dv}{dt} = -(g + kv^2)$, so

$$T = \int_0^u \frac{dv}{g + kv^2} = \frac{1}{k} \cdot \frac{1}{\sqrt{g/k}} \left[\tan^{-1} \frac{v}{\sqrt{g/k}} \right]_0^u = \frac{1}{\sqrt{gk}} \tan^{-1} \left(u \sqrt{\frac{k}{g}} \right).$$

(The question is sequenced to force **both** acceleration forms — the marking-centre theme of 2020, 2021, 2023, 2024, 2025.)

(a)(iii) Falling: $\ddot{x} = g - kv^2$ (resistance now opposes the downward motion), so the terminal velocity is $v_T = \sqrt{g/k}$, where the acceleration vanishes. While falling, $v < v_T$ always: the speed increases towards v_T but the acceleration $g - kv^2 \rightarrow 0$ as $v \rightarrow v_T$, so v_T is approached asymptotically and never attained in the finite fall from height H . Hence the landing speed is less than $\sqrt{g/k}$.

(b)(i) $|-8 + 8\sqrt{3}i| = 16$, $\arg = \frac{2\pi}{3}$. The fourth roots have modulus $16^{1/4} = 2$ and arguments $\frac{2\pi/3 + 2k\pi}{4} = \frac{\pi}{6} + \frac{k\pi}{2}$:

$$z = 2e^{i\pi/6}, \quad 2e^{2i\pi/3}, \quad 2e^{-5i\pi/6}, \quad 2e^{-i\pi/3}.$$

(b)(ii) Four points on the circle $|z| = 2$, equally spaced at right angles — the vertices of a square centred at O .

(b)(iii) $2e^{i\pi/6} = 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \sqrt{3} + i$.

(c) (i) $\bar{z}$: reflect z in the real axis — fourth quadrant, same modulus (outside the unit circle). (ii) $i\bar{z}$: rotate $\bar{z}$ anticlockwise by $\frac{\pi}{2}$ — argument $\frac{\pi}{2} - \arg z \approx 50^\circ$, same modulus. (iii) $\frac{z^2}{|z|}$: argument doubles to

$\approx 80^\circ$, modulus $\frac{|z|^2}{|z|} = |z|$ — the same distance from O as z , rotated to twice the argument. (1 mark each; the marks are for correct modulus **and** argument reasoning, not computation.)

Question 16

(a)(i) $|z + w|^2 + |z - w|^2 = (z + w)(\bar{z} + \bar{w}) + (z - w)(\bar{z} - \bar{w}) = 2z\bar{z} + 2w\bar{w} = 2|z|^2 + 2|w|^2$ (the cross terms $z\bar{w} + w\bar{z}$ cancel).

(a)(ii) With $|z| = |w| = 1$, part (i) gives $|z + w|^2 + |z - w|^2 = 4$. Suppose, for contradiction, that $|z + w| < \sqrt{2}$ **and** $|z - w| < \sqrt{2}$. Then $|z + w|^2 + |z - w|^2 < 2 + 2 = 4$ — contradicting the identity. Hence at least one of them is $\geq \sqrt{2}$. (1 mark the assumption stated, 1 mark the contradiction exhibited and the conclusion written.)

(a)(iii) $z = 1, w = i$: $|1 + i| = |1 - i| = \sqrt{2}$, and $|z| = |w| = 1$. (Any perpendicular pair on the unit circle works.)

(b)(i) $v \frac{dv}{dx} = -(c + kv)$, so

$$D = \int_0^u \frac{v dv}{c + kv} = \int_0^u \left(\frac{1}{k} - \frac{c/k}{c + kv} \right) dv = \frac{u}{k} - \frac{c}{k^2} \ln \frac{c + ku}{c} = \frac{u}{k} - \frac{c}{k^2} \ln \left(1 + \frac{ku}{c} \right).$$

(1 mark the correct acceleration form and separation, 1 mark the division/decomposition of $\frac{v}{c+kv}$, 1 mark limits to the given form.)

(b)(ii) $\frac{dv}{dt} = -(c + kv)$: $T = \int_0^u \frac{dv}{c + kv} = \frac{1}{k} \ln \left(1 + \frac{ku}{c} \right).$

(c)(i) With $u = \pi - x$ (so $\sin x = \sin u$, $\cos x = -\cos u$, $dx = -du$, limits swap):

$$I = \int_0^\pi \frac{(\pi - u) \sin u}{1 + \cos^2 u} du = \pi \int_0^\pi \frac{\sin u}{1 + \cos^2 u} du - I,$$

so $2I = \pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx$. (1 mark the substitution with both limits, 1 mark collecting I .)

(c)(ii) Let $c = \cos x$, $dc = -\sin x dx$: $\int_{-1}^1 \frac{dc}{1 + c^2} = [\tan^{-1} c]_{-1}^1 = \frac{\pi}{2}.$

(c)(iii) $I = \frac{\pi}{2} \cdot \frac{\pi}{2} = \frac{\pi^2}{4}.$

Prediction provenance

Which consensus prediction each part of this paper realises. Full evidence panels:
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Paper item	Prediction	Agreement
MC Q1, Q2 contrapositive + quantifier negation	ext2-q1-logic-mc	0.82 · 6 (all)
Q11(a), (b), (f) conjugate arithmetic, realised quotient, exponential-form 7th power	ext2-q2-q11-complex-opener	0.81 · 5 (fable, opus, grok, deepseek, gpt-5.6-sol)
Q12(a) + MC Q8 partial fractions, $(x - p)(x^2 + q) \rightarrow \ln + \arctan$	ext2-q3-partial-fractions	0.78 · 6 (all)
Q12(c) + MC Q6 SHM from v^2 quadratic, shifted centre; amplitude-off-constant-term distractor	ext2-q4-shm	0.76 · 6 (all)
Q13(a) reduction formula by parts with "hence" evaluation	ext2-q5-reduction-formula	0.71 · 6 (all) — realised as the predicted reversion to integration by parts
Q11(c), (d) + MC Q4 angle between vectors, line through two points, point-on-line test	ext2-q6-vector-line-toolkit	0.71 · 5 (fable, opus, grok, gpt-5.6-sol, deepseek)
Q13(c) AM–GM prove-then-apply, chained 3-variable inequality	ext2-q7-am-gm	0.66 · 5 (opus, fable, grok, gpt-5.6-sol, deepseek)
Q13(b) Argand region: bisector inequality $\cap$ disc, dashed/solid conventions	ext2-q8-argand-region	0.64 · 6 (all)
Q15(a) vertical resisted motion kv^2 , sequenced to force both acceleration forms	ext2-q9-resisted-vertical	0.64 · 5 (deepseek, gpt-5.6-sol, grok, fable + opus variant)
Q11(e) contradiction: irrationality of $\log_2 3$ (logical frame marked)	ext2-q10-contradiction-irrationality	0.63 · 6 (all) — type consensus; fresh surface constant
Q14(a) vector ratio proof by equating coefficients of non-parallel vectors	ext2-q11-vector-ratio-proof	0.62 · 5 (fable, opus, grok, gpt-5.6-sol, deepseek)
Q12(b) $t = \tan \frac{x}{2}$ supplied-substitution integral; Q16(c) the $u = a - x$ symmetry variant	ext2-q12-substitution-integral	0.60 · 5 — both panel sub-variants realised
Q14(b) De Moivre $\rightarrow \sin 5\theta$ polynomial $\rightarrow$ exact surd product	ext2-q13-de-moivre-exact-value	0.58 · 5 — fresh surface (sine identity) per the corpus-echo note on $\cos 5\theta$
Q12(d) inequality induction with base case $n = 5$, auxiliary inequality cited	ext2-q14-induction-slot	0.52 · 6 (all, type-level) — inequality form favoured over the divisibility outsider; the DeepSeek/Grok shared surface ($7^n - 2^n$) was a training-data echo and was not used
Q13(d) shortest distance from point to line via foot of perpendicular	watch list	$\sim 0.63 \cdot 3$ (deepseek, opus, gpt-5.6-sol) — the panel's sharpest split: fable and grok rest it
Q15(b) $z^4 = c$ for non-real c , equally spaced roots, Argand plot	watch list	$\sim 0.60 \cdot 3$ — fresh surface (2025 used $z^5 + 1 = 0$)

Paper item	Prediction	Agreement
Q16(b) horizontal resisted motion, constant-plus- v , distance-to-rest log form	watch list	$\sim 0.63 \cdot 2$ (opus, fable)
MC Q5 $a = v \, dv/dx$ with the omitted- v distractor	watch list	$\sim 0.60 \cdot 2$ (grok, gpt-5.6-sol) + deepseek trend
Q14(c) + MC Q7 conjugate-root polynomial, remaining zeros by sum/product	watch list	$\sim 0.54 \cdot 3$ (fable, gpt-5.6-sol, grok)
Q14(d) complex-coefficient quadratic via Cartesian $\sqrt{\Delta}$	watch list	$\sim 0.48 \cdot 3$ (deepseek, fable, opus) — fable's bold call: five-year Section II coverage gap in the syllabus's final year
Q16(a) abstract bound proof: parallelogram identity + contradiction on moduli	watch list	$0.65 \cdot$ fable (specific + structural; 2021 16(a), 2022 15(d), 2025 16(a) lineage)
Q15(c) Argand transformation plot: $\bar{z}$, $i\bar{z}$, $z^2/ z $ without computation	watch list	$\sim 0.55 \cdot 2$ (fable, gpt-5.6-sol)
MC Q3 principal argument; MC Q9 De Moivre power; MC Q10 bisector locus	topic-level consensus	— · —
Not realised: resisted projectile (rested by fable and grok after 2025 16(b)); volumes-of-revolution revival (grok alone, 0.48)	topic-level consensus	— · —

Built by an AI panel and backtested against the hidden 2025 papers before publication — method at intuitioneducation.com.au/hsc-2026/how-we-did-it. Scored publicly in November 2026. Independent Intuition Education material from publicly available NESA documents; not affiliated with or endorsed by NESA.