INTUITION EDUCATION

HSC 2026 · PREDICTED PRACTICE PAPER

2026 HSC Mathematics Extension 1

Original practice material — not a NESA paper

Total marks: 70

GENERAL INSTRUCTIONS

- Reading time — 10 minutes
- Working time — 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Section I — 10 marks (Questions 1–10)
Attempt Questions 1–10. Allow about 15 minutes for this section.

Section II — 60 marks (Questions 11–14)
Attempt Questions 11–14. Allow about 1 hour and 45 minutes for this section.

HOW THIS PAPER WAS BUILT

This full-length practice paper was synthesised from a six-model AI consensus (Fable, Opus, GPT-5.6 Sol, Gemini 3.1 Pro, Grok, DeepSeek) over 152 question-level predictions for the 2026 HSC Mathematics Extension 1 examination. Every consensus cluster with agreement probability ≥ 0.55 is realised as a question below, with the remaining marks drawn from the panel's watch list; all questions are original Intuition Education compositions in NESA style — no question is reproduced from a past HSC paper. A prediction provenance table follows the marking notes.

Answers and marking notes are published as a separate PDF — sit the paper first, then download the solutions from intuitioneducation.com.au/hsc-2026/maths-extension-1.

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Section I

10 marks — Attempt Questions 1–10 — Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

Question 1

How many distinct arrangements can be made using all the letters of the word **PARALLEL**?

A. 1680

B. 6720

C. 3360

D. $40\,320$

Question 2

The projection of $\underset{\sim}{u}$ onto $\underset{\sim}{v}$ is given by $\dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{v}|^2}\,\underset{\sim}{v}$.

What is the projection of $\underset{\sim}{u} = 3\underset{\sim}{i} - \underset{\sim}{j}$ onto $\underset{\sim}{v} = \underset{\sim}{i} + 2\underset{\sim}{j}$?

A. $\dfrac{1}{10}\left(3\underset{\sim}{i} - \underset{\sim}{j}\right)$

B. $\dfrac{1}{5}\left(3\underset{\sim}{i} - \underset{\sim}{j}\right)$

C. $\dfrac{1}{5}\left(\underset{\sim}{i} + 2\underset{\sim}{j}\right)$

D. $\dfrac{1}{10}\left(\underset{\sim}{i} + 2\underset{\sim}{j}\right)$

Question 3

What is the value of $\cos^{-1}\left(\cos\dfrac{5\pi}{4}\right)$?

A. $\dfrac{\pi}{4}$

B. $\dfrac{3\pi}{4}$

C. $\dfrac{5\pi}{4}$

D. $-\dfrac{3\pi}{4}$

Question 4

Using the substitution $t = \tan\dfrac{x}{2}$, the expression $\dfrac{1-\cos x}{\sin x}$ simplifies to which of the following?

A. t

B. $\dfrac{1}{t}$

C. $\dfrac{2t}{1-t^2}$

D. t^2

Question 5

How many distinct solutions are there to the equation $\sin 2x = \sin x$ for $0 \le x \le 2\pi$?

A. 3

B. 4

C. 5

D. 6

Question 6

Air is pumped into a spherical balloon so that its volume increases at a constant rate of $100\ \text{cm}^3\ \text{s}^{-1}$.

What is the rate of increase of the radius, in cm s^{-1}, when the radius is 5 cm?

A. $\dfrac{1}{\pi}$

B. $\dfrac{4}{\pi}$

C. $\dfrac{1}{4\pi}$

D. $\dfrac{100}{\pi}$

Question 7

A sample of 100 voters is taken from a large population in which the proportion supporting a proposal is $p = 0.2$. Let $\hat{p}$ denote the sample proportion.

What is the standard deviation of $\hat{p}$?

A. 0.0016

B. 0.004

C. 0.4

D. 0.04

Question 8

What is the coefficient of x^4 in the expansion of $(x+2)^7$?

A. 560

B. 280

C. 140

D. 70

Question 9

Let $f(x) = x^3 + 3x + 1$. The function f is increasing for all real x, so the inverse function f^{-1} exists.

What is the value of $\left(f^{-1}\right)'(5)$?

A. $-\dfrac{1}{6}$

B. $\dfrac{1}{5}$

C. 6

D. $\dfrac{1}{6}$

Question 10

What is the smallest number of people needed in a room to guarantee that at least 4 of them were born on the same day of the week?

A. 21

B. 22

C. 25

D. 28

Section II

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60 marks — Attempt Questions 11–14 — Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available. For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) — Use the Question 11 Writing Booklet *(15 marks)*

(a) Let $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = -\underset{\sim}{i} + 4\underset{\sim}{j}$.

(i) Find $2\underset{\sim}{a} - \underset{\sim}{b}$. **(1 mark)**

(ii) Find $\underset{\sim}{a} \cdot \underset{\sim}{b}$. **(1 mark)**

(b) The polynomial $P(x) = 2x^3 + ax^2 + bx - 3$ has $(x-1)$ as a factor. When $P(x)$ is divided by $(x+2)$, the remainder is -3.

Find the values of a and b. **(3 marks)**

(c) Solve $\dfrac{3}{x} \leq x + 2$. **(3 marks)**

(d)

(i) Express $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. **(2 marks)**

(ii) Hence, or otherwise, solve $\sqrt{3}\sin x - \cos x = \sqrt{2}$ for $0 \leq x \leq 2\pi$. **(2 marks)**

(e) Use the substitution $u = x^2 + 9$ to evaluate

$$\int_0^4 x\sqrt{x^2+9}\,dx.$$

(3 marks)

Question 12 (15 marks) — Use the Question 12 Writing Booklet *(15 marks)*

(a) Use mathematical induction to prove that $5^n + 2 \times 11^n$ is divisible by 3 for all integers $n \geq 1$. **(3 marks)**

(b) Solve the differential equation

$$\frac{dy}{dx} = \frac{x}{y},$$

given that $y = -4$ when $x = 3$. Give your answer with y as the subject, justifying your choice of sign. **(3 marks)**

(c) The diagram shows the direction field for the differential equation

$$\frac{dy}{dx} = \frac{1}{2}\, y\,(2 - y).$$

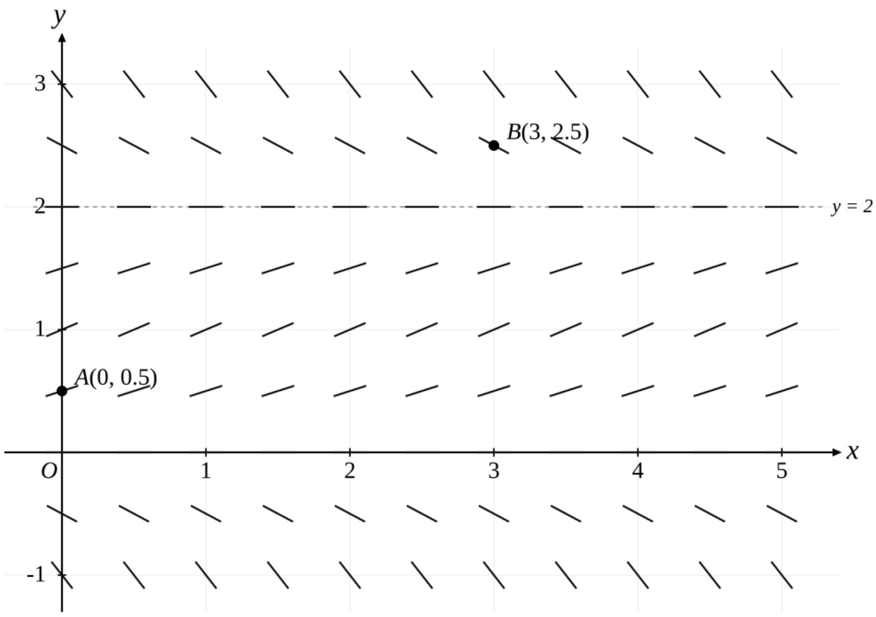

DIAGRAM PROVIDED IN THE EXAM

(i) On the diagram, sketch the graph of the particular solution that passes through the point $A(0, 0.5)$, showing its behaviour as x increases. **(2 marks)**

(ii) Explain why the solution through A can never pass through the point $B(3, 2.5)$. **(1 mark)**

(d) The region bounded by the curve $y = \dfrac{4}{x^2+1}$, the y-axis and the line $y = 1$ is rotated about the y-axis.

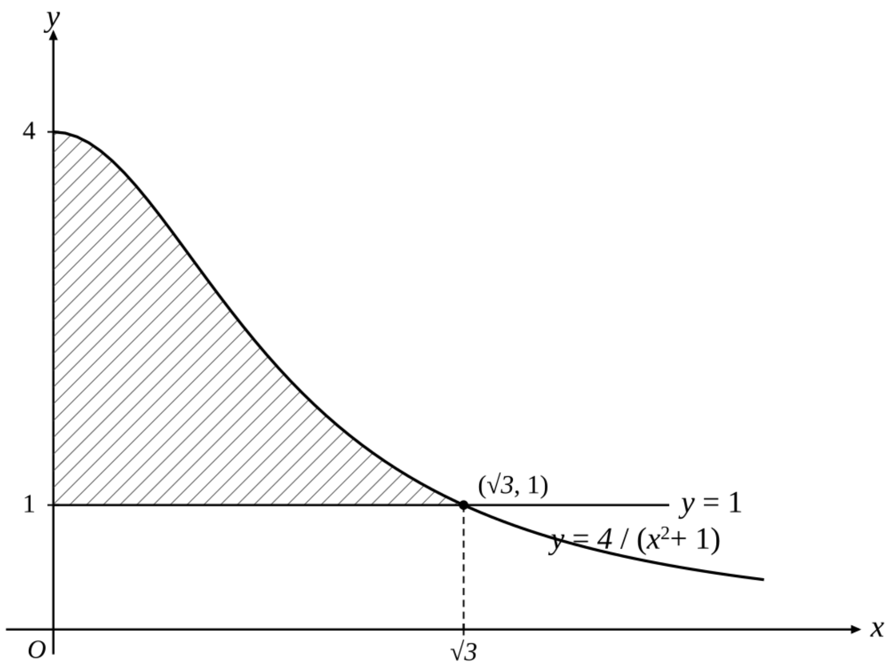

DIAGRAM PROVIDED IN THE EXAM

Find the exact volume of the solid formed. **(3 marks)**

(e) Solve $\cos 2x + \cos x + 1 = 0$ for $0 \leq x \leq 2\pi$. **(3 marks)**

Question 13 (14 marks) — Use the Question 13 Writing Booklet *(14 marks)*

(a) Sketch the graph of $y = 3\cos^{-1}\left(\dfrac{x}{2}\right)$, clearly labelling the coordinates of the endpoints of the graph. **(2 marks)**

(b) The points A and B lie at opposite ends of a diameter of a circle with centre O and radius r, and P is any other point on the circle. Relative to O, the points A, B and P have position vectors $\underset{\sim}{a}$, $-\underset{\sim}{a}$ and $\underset{\sim}{p}$ respectively.

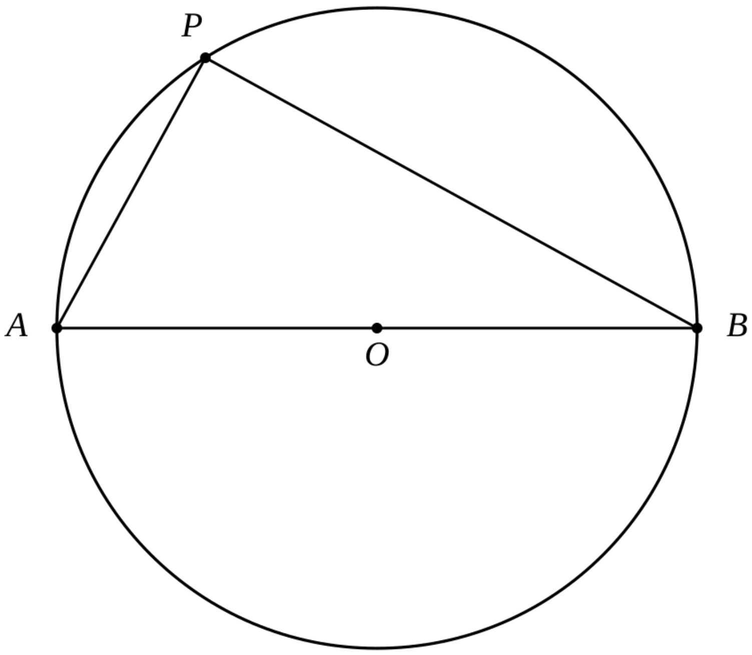

DIAGRAM PROVIDED IN THE EXAM

Using vectors, prove that $\angle APB = 90^\circ$. **(3 marks)**

(c) A large nursery plants 400 seeds. Each seed germinates with probability 0.9, independently of all other seeds. The number X of seeds that germinate is approximated by a normal distribution.

The table gives values of $P(Z \leq z)$ for a standard normal variable Z.

z	1.3	1.4	1.5	1.6	1.7
$P(Z \leq z)$	0.9032	0.9192	0.9332	0.9452	0.9554

(i) Find the mean and standard deviation of X. **(1 mark)**

(ii) The nursery's records show that the probability that **fewer than** k seeds germinate is approximately 0.0668. Using the normal approximation, find the value of k. **(3 marks)**

(d) Evaluate

$$\int_0^{\frac{\pi}{8}} \sin^2(2x)\ dx,$$

giving your answer in exact form. **(3 marks)**

(e) The equation $x^3 + 2x^2 - 3x - 1 = 0$ has roots α, β and γ.

Find the value of $\alpha^2 + \beta^2 + \gamma^2$. **(2 marks)**

Question 14 (16 marks) — Use the Question 14 Writing Booklet *(16 marks)*

(a) By first completing the square in the denominator, show that

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \frac{\pi}{8}.$$

(3 marks)

(b) Five points are placed inside or on the boundary of an equilateral triangle of side length 2 cm.

By dividing the triangle into suitable regions, prove that at least two of the points must be no more than 1 cm apart. **(3 marks)**

(c) You may use the identity $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$.

(i) Show that $t = \tan\dfrac{\pi}{12}$, $t = \tan\dfrac{5\pi}{12}$ and $t = \tan\dfrac{3\pi}{4}$ are the three roots of the equation

$$t^3 - 3t^2 - 3t + 1 = 0.$$

(2 marks)

(ii) Hence show that $\tan\dfrac{\pi}{12} + \tan\dfrac{5\pi}{12} = 4$. **(1 mark)**

(d) Two balls are launched from a point O on horizontal ground. Take $g = 10 \text{ m s}^{-2}$ and ignore air resistance.

Ball A is launched at time $t = 0$ with speed $V \text{ m s}^{-1}$ at an angle of 60° above the horizontal. Its position at time $t \geq 0$ seconds is given by

$$\underset{\sim}{r}_A(t) = \frac{Vt}{2}\,\underset{\sim}{i} + \left(\frac{\sqrt{3}\,Vt}{2} - 5t^2\right)\underset{\sim}{j}. \qquad \text{(Do NOT prove this.)}$$

Ball B is launched from O at time $t = T$, where $T > 0$, with the same speed $V \text{ m s}^{-1}$ but at an angle of 30° above the horizontal. Its position at time $t \geq T$ seconds is given by

$$\underset{\sim}{r}_B(t) = \frac{\sqrt{3}\,V(t-T)}{2}\,\underset{\sim}{i} + \left(\frac{V(t-T)}{2} - 5(t-T)^2\right)\underset{\sim}{j}. \qquad \text{(Do NOT prove this.)}$$

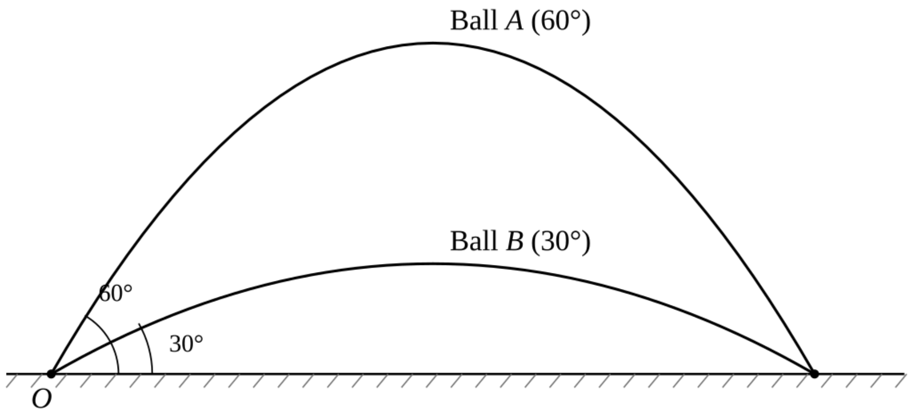

DIAGRAM PROVIDED IN THE EXAM

(i) Show that ball A lands at time $t = \dfrac{\sqrt{3}\,V}{10}$, and that ball B is in flight for $\dfrac{V}{10}$ seconds. Hence show that the two balls land at the same point. **(3 marks)**

(ii) The two balls land at the same instant. Show that $T = \dfrac{(\sqrt{3}-1)V}{10}$, and hence find the exact value of V for which $T = 2$. **(2 marks)**

(iii) Show that, at the moment the balls land, the acute angle between their velocity vectors is $\dfrac{\pi}{6}$. **(2 marks)**

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